A 1 2 3 4

Total outcomes $3^4 = 81$

① B never receives

$2^4 = 16$

② B touches only once

at 1^{st} position, $1 \times 3 \times 2 \times 2$

2^{nd}, $2 \times 1 \times 3 \times 2$

3^{rd} $2 \times 2 \times 1 \times 3$

4^{th} $2 \times 2 \times 2 \times 1$

total 44

③ at lest twice : $81 - 16 - 44 = 21$

④ Prob : $\frac{21}{81} = \frac{7}{27}$

Q2

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots \frac{99}{100!}$$

$$= \left(\frac{2}{2!} - \frac{1}{2!}\right) + \left(\frac{3}{3!} - \frac{1}{3!}\right) + \cdots + \left(\frac{100}{100!} - \frac{1}{100!}\right)$$

$$= \left(1 + \frac{1}{2!} + \cdots + \frac{1}{99!}\right) - \left(\frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{100!}\right)$$

$$= 1 - \frac{1}{100!}$$

So, $x = 101!\left(1 - \frac{1}{100!}\right)$

$$101! - x = 101!\left(1 - 1 + \frac{1}{100!}\right)$$

$$= 101! \cdot \frac{1}{100!}$$

$$= 101.$$

Q3

$$x + \frac{1}{x} = 5$$

Goal: find $x^4 + \frac{1}{x^4}$.

$\triangleright$ $x^2 + 2 + \frac{1}{x^2} = 25$

$$x^2 + \frac{1}{x^2} = 23$$

$$x^4 + 2 + \frac{1}{x^4} = 23^2$$

$$\therefore \quad x^4 + \frac{1}{x^4} = 527$$

$$\begin{array}{r} 23 \\ \times 23 \\ \hline 69 \\ 46 \\ \hline 529 \end{array}$$

Q4

$\frac{4}{9}$ — prob. of winning

$\frac{2}{9}$ — prob. of winning at rains

$\frac{3}{5}$ — prob. of rainy days

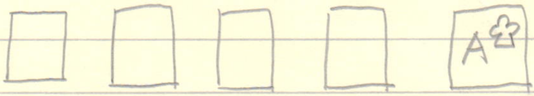
A: rainy day B: Sparrows wins

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$= \frac{\frac{2}{9} \cdot \frac{3}{5}}{\frac{4}{9}}$$

$$= \frac{6}{20} = \frac{3}{10}$$

Q5



① The same rank cards are Aces

$\binom{48}{1} = 48$, choices of the other card.

② The same rank cards are not Aces.

$\binom{12}{1} = 12$, choose a set and own all cards.

60 possible hands

Q6

n bits. hamming distance d.

choose d from n.

$\binom{n}{d}$

A monotonic increasing sequence of length 5
Given any 5 digits, only 1 way to satisfy the constraints.

Suppose x_i: the number of times that digit i is used. Then

$$x_0 + x_1 + \cdots x_9 = 5$$

Solutions to the above equation is the number of possible codes.

e.g.

+ | + + + | + + | + + | + |

$x_1 = 1 \quad x_4 = 1 \quad x_6 = 1 \quad x_8 = 1 \quad x_9 = 1$

The code is 14689

We have 5 ones, but 9 "+" to fill. So total possible combs is $\binom{9+5}{5} = 2002$

Another Example

0 1 2 3 4 5 6 7 8 9

+ + / / + + + / + + / + / +

$x_2=2$, $x_5=1$ $x_7=1$ $x_8=1$

so the code is 22578.